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EHiXs

A new parallel Code for Higgs Production

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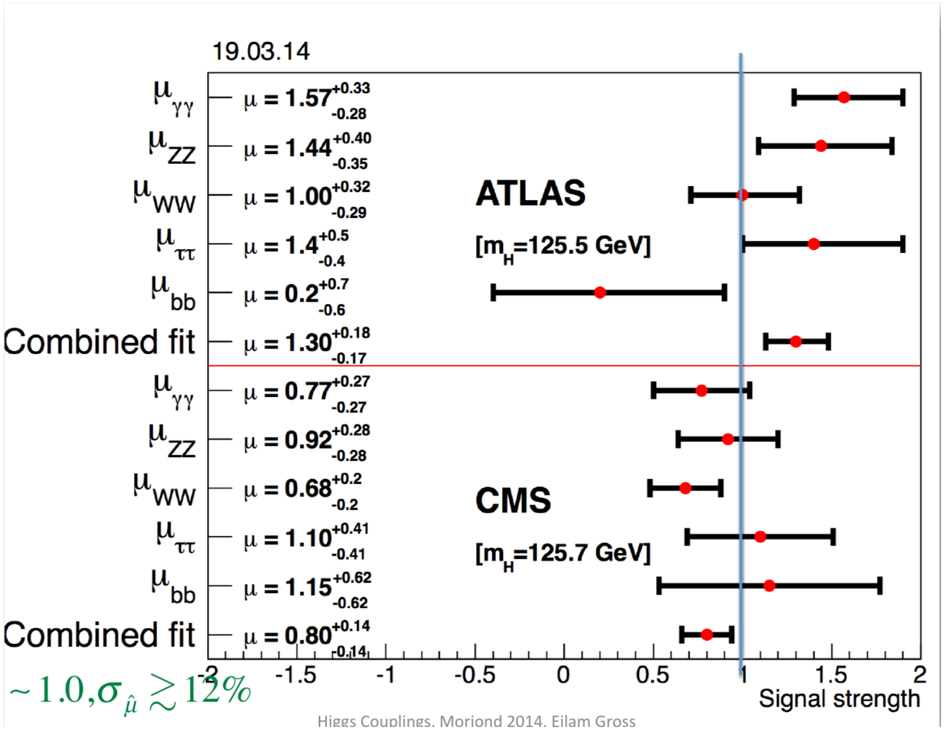
One of the major achievement of the LHC is the measurement of mass and couplings of the Higgs boson.



These measurements require accurate theoretical predictions for the fully differential Higgs boson cross section.

Some Public Tools for SM Higgs Production

- HNNLO differential fixed order QCD NNLO, NLO EW,.. [Catani, Grazzini]
- FeHiPro differential fixed order NNLO QCD, NLO EW, .. [Anastasiou, Bucherer, Lazopoulos, Stoeckli]
- Hqt differential in pt re-summed To NNLL [Bozzi, Catani, de Florian, Grazzini, Ferrera]
- HRes differential threshold resummation for small pt [De Florian, Ferrera, Grazzini, Tommasini]
- Powheg differential NLO matched to parton shower [Alioli, Nason, Oleari, Re]
- MC@NLO differential NLO matched to parton shower [Frixione, Weber]
- Peter differential in pt re-summed To NNNLL with SCET [Becher, Lorentzen, Schwartz]
- JetVHeto differential in jet veto resummed NNLL [Banfi, Salam,Zanderighi]
- Higlu inclusive fixed order NLO exact [Spira]
- IHiXS inclusive fixed order NNLO QCD, NLO EW,.. [Anastasiou, Buehler, FH, Lazopoulos]
- ggh@nnlo inclusive fixed order NNLO QCD [Harlander, Kilgore]
- ...

General state of available tools is good.

FeHiPro is no longer maintained!

- Code is patched together from several different sources
- Difficult to mofify

To have at least two maintained public fully differential NNLO event generators, we are now working on a new code:

eHiXS

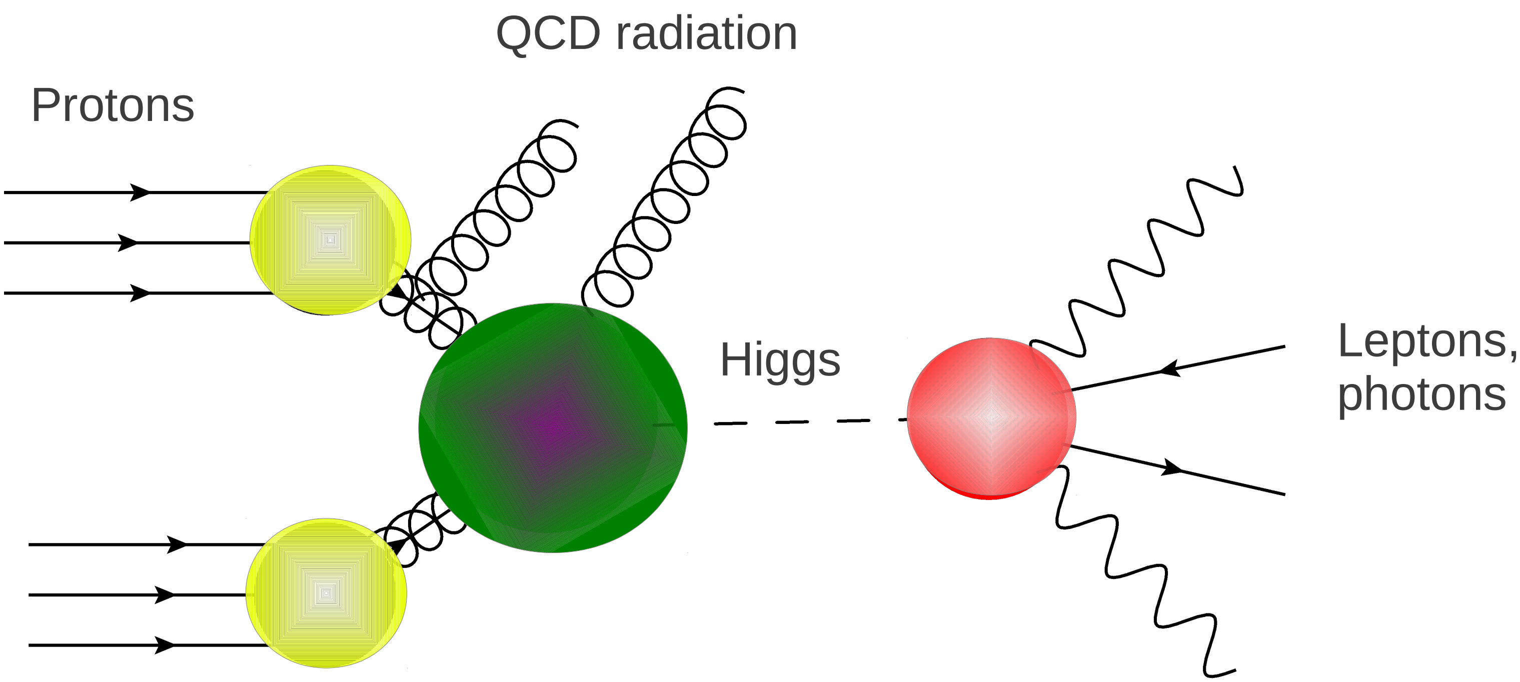
exclusive Higgs Cross-section

- Flexible framework
  - Written in C++
  - Can easily add further corrections

- User friendly
  - Straight forward to define arbitrary numbers of new observables, final state cuts, jet algorithms, ..

- It's Parallel
  - It can use all cores on your laptop, or run on several 100 cores on a cluster

What is inside eHiXs?



Production
QCD exact NLO
QCD effective NNLO
EW 2-loop
EW real
Mixed QCD EW
bb->H
.

Decays at LO
H→WW→llll
H→ZZ→llll
H →Zγ→llγ
H→γγ

The most time consuming part of a fully differential Higgs Monte Carlo at NNLO is the **Double Real Emission.**

A fast code therefore requires a fast implementation of the double real!

Several methods on the market:

- Sector Decomposition
- qt-subtraction
- Antenna subtraction
- ..

Here we use yet another method:

- Non-linear Mappings & Integrand reduction

Non-linear Mappings & Integrand Reduction

$$\sigma[J] = \int \mathrm{d}\Phi |\mathcal{A}(\{p_i\})|^2 J(\{p_i\})$$

1) Laurent expand Integrand

$$|\mathcal{A}|^2 = \sum_i \sum_{\sigma \in G_i} F_i(\{p_\sigma\}) \;\Rightarrow \sigma[J] = \sum_i \int \mathrm{d}\Phi F_i(\{p_i\}) \sum_{\sigma \in G_i} J(\{p_\sigma\})$$

F_i is a function with reduced singularity structure. G_i is a group of permutations.

2) Transform to a parameterisation which factorises singularities (use projective transformations)

$$\int d\Phi F_i = \int_0^1 \left(\prod_i \frac{\mathrm{d}\lambda_i}{\lambda_i^{1+a_{ij}\epsilon}} \right) f_j(\lambda)$$

3) Automatic recursive construction of IR counterterms and isolation of poles:

$$\int_0^1 \frac{d\lambda}{\lambda^{1+a\epsilon}} f(\lambda, ..) = \frac{f(0, ..)}{a\epsilon} + \sum_{k=0}^{\infty} \frac{(-a\epsilon)^k}{k!} \int_0^1 d\lambda \frac{\log^k \lambda}{\lambda} \big(f(\lambda, ..) - f(0, ..) \big)$$

Integrand Reduction

$$|\mathcal{A}|^2 = \text{sum of Cut Diagrams} = \sum_j \frac{N_j(S)}{\prod_{i \in S_j} D_i^{n_i}}$$

Assume basis $S = \{s_1, .., s_n\}$ such that $D_i = \sum_j c_{ij} s_j$

Denominators span a subspace

$$S_j = \{D_1, .., D_k\}$$

Split full basis into subspace and quotient space

$$S = S_j \cup S/S_j = \{D_1, .., D_k, x_{k+1}, .., x_n\}$$

Allows to perform a „polynomial division“ [Yang Zhang , Mastrolia]

$$N_j(S) = \sum_{n_1..n_k} C_{n_1..n_k}(S/S_j) D_1^{n_1}..D_k^{n_k}$$

Recursive application of polynomial division allows to arrive a Laurent expansion

$$|\mathcal{A}|^2 = \sum_j \frac{\mathcal{N}_j(S/S_j)}{\prod_{i \in S_j} D_i}$$

The „residues“ $\mathcal{N}_j$ are not unique! But depend on the choice of the quotient basis S/S_j

Or in other words the order of multivariate division.

Enforce Discrete Symmetries

Consider permutations which leave the full integrand invariant:

$$\frac{1}{s_{12}s_{34}} \xrightarrow{1\leftrightarrow 3} \frac{1}{s_{23}s_{14}}$$

Permutation relating different denominators

$$\frac{1}{s_{12}s_{34}} \xrightarrow{1\leftrightarrow 2} \frac{1}{s_{12}s_{34}}$$

Permutation leaving denominators invariant

Choose the x_j such that they live in a representation of the symmetry group.
Then the residues satisfy all the right symmetry properties

$$\begin{array}{c} \sigma \\ S_1 \longrightarrow S_2 \\ \{x_k^{(1)}\} \longrightarrow \{x_k^{(2)}\} \\ \mathcal{N}_1 \longrightarrow \mathcal{N}_2 \end{array} \qquad \begin{array}{c} \sigma \\ S_1 \longrightarrow S_1 \\ \{x_k^{(1)}\} \longrightarrow \{x_k^{(1)}\} \\ \mathcal{N}_1 \longrightarrow \mathcal{N}_1 \end{array}$$

In other words. Impose symmetry properties on the Groebner basis of the quotient space

Factor out the sum over Symmteries:

$$|\mathcal{A}|^2 = \sum_{j=1}^{n_S} \frac{\mathcal{N}_j(\{x_k^{(j)}\})}{\prod_{i\in S_j} D_i} = \sum_{j=1}^{\sim n_S/D_G} \sum_{\sigma\in G_j} \underbrace{\frac{\mathcal{N}_j(S/\sigma(S_j))\})}{\prod_{i\in\sigma(S_j)} D_i}}_{= F_j}$$

Use that the phase space measure is invariant under permutations

$$\int d\Phi J(\{p_i\}) \sum_{\sigma\in G} F(\{p_i\}) = \int d\Phi F(\{p_i\}) \sum_{\sigma\in G} J(\{p_i\})$$

Can gain a potentially large Symmetry factor in evaluation time

$$\Rightarrow \sigma[J] = \sum_i \int \mathrm{d}\Phi F_i(\{p_i\}) \sum_{\sigma\in G_i} J(\{p_\sigma\})$$

Can we always find such a basis for the residues?

$$S = \{s_{12}, s_{34}, s_{23}, s_{14}, s_{13}, s_{24}\}$$

For 2-particle denominators this is is always obvious

$$\frac{\mathcal{N}(s_{23}, s_{14})}{s_{12}s_{34}s_{13}s_{24}} + \frac{\mathcal{N}(s_{13}, s_{24})}{s_{12}s_{34}s_{23}s_{14}} + \frac{\mathcal{N}(s_{12}, s_{34})}{s_{23}s_{14}s_{13}s_{24}}$$

For 3-particle denominators can use squares of asymmetric combinations

$$\frac{\mathcal{N}(s_{12}, (s_{23} - s_{24})^2, s_{13}, s_{14})}{(s_{23} + s_{24} + s_{34})s_{34}} + \frac{\mathcal{N}(s_{12}, (s_{13} - s_{14})^2, s_{23}, s_{24})}{(s_{13} + s_{14} + s_{34})s_{34}}$$

Most complicated at NNLO
is the ggggH squared Amplitude.

Contains 351 interferences.

$$|M_{H\to gggg}|^2 = \frac{1}{64}N^2(N^2-1)\sum_{\sigma\in S_4} F_{H\to gggg}(p_{\sigma_1},p_{\sigma_2},p_{\sigma_3},p_{\sigma_4})$$

Exhibits several useful properties:

- Symmetries are manifest.

- Worst singularities have been isolated.

- Spurious (quadratic) singularities have been cancelled.

Remains to integrate different singularity structures

$$\begin{aligned}
F_{H\to gggg}(p_1,p_2,p_3,p_4) = {}& -1256 - 72D^2 + 740D + 8\frac{\left(s_{23}{}^2+s_{14}s_{23}+s_{14}{}^2\right)^2(D-2)}{s_{12}s_{13}s_{24}s_{34}} \\
&+8\frac{(D-2)^2\left(-s_{14}s_{23}+s_{13}s_{24}\right)^2}{s_{12}{}^2s_{34}{}^2} + 4\frac{(D-2)^2\left((-s_{23}+s_{24})\,s_{134}+s_{234}\,(s_{13}-s_{14})\right)^2 m_H{}^4}{s_{134}{}^2s_{34}{}^2s_{234}{}^2} \\
&+8\frac{(D-2)\,(D-4)\,m_H{}^2\left((-s_{14}+s_{24})\,s_{123}+s_{124}\,(s_{13}-s_{23})\right)^2}{s_{34}s_{124}s_{123}s_{12}{}^2} \\
&+32\frac{\left(s_{24}{}^2-s_{13}s_{24}+s_{13}{}^2\right)^2(D-2)}{s_{234}s_{124}s_{14}s_{34}} + 24\frac{(D-2)^2\,m_H{}^4}{s_{124}{}^2} + 8/3\frac{\left(36D^2+89D-418\right)m_H{}^2}{s_{234}} \\
&+64\frac{\left(\left(-7D+14+D^2\right)s_{13}{}^2-2\,(D-2)\,s_{24}s_{13}+(D-2)\,s_{24}{}^2\right)m_H{}^4}{s_{134}s_{12}s_{34}s_{123}} \\
&+32\frac{\left(s_{12}\,(s_{14}+s_{12}+s_{23})\,(D-2)+(7D-20)\,s_{14}s_{23}+2\left(s_{14}{}^2+s_{23}{}^2\right)(D-2)\right)m_H{}^2}{s_{13}s_{24}s_{34}} \\
&-\frac{768\,(D-2)\,m_H{}^2+2\left(244-249D+42D^2\right)s_{34}-(s_{24}+s_{13}+s_{23}+s_{14})\left(-237D+126D^2+487\right)}{s_{12}} \\
&-32\frac{m_H{}^2\left(s_{13}{}^2+s_{14}{}^2+s_{23}{}^2+s_{24}{}^2-2\,(s_{14}+s_{24})\,(s_{13}+s_{23})+2s_{13}s_{23}+2s_{14}s_{24}+4m_H{}^4\right)(D-2)}{s_{34}s_{234}s_{134}} \\
&-\frac{8}{s_{13}s_{24}}\Big[\left(s_{12}{}^2+s_{14}{}^2+s_{23}{}^2+s_{34}{}^2\right)\left(4D+D^2-14\right)+2\left(s_{12}{}^2+s_{23}{}^2+s_{24}{}^2\right)(D-2) \\
&+6\,(D-2)\,s_{24}s_{12}+8s_{23}\,(s_{12}+s_{24})\,(D-2)\Big] + \frac{32}{s_{13}s_{34}}\Big[(-24+11D)\,s_{14}s_{23} \\
&+2s_{14}\,(s_{12}+s_{24})\,(-9+4D)+2\,(D-3)\,s_{14}{}^2-2\,(s_{23}+s_{14})\,(s_{12}+s_{34})\left(-10D+26+D^2\right) \\
&-(s_{14}s_{23}+s_{12}s_{34})\left(6D^2+94-41D\right)\Big] + \frac{2}{s_{234}s_{24}}\Big[\left(s_{23}{}^2+s_{34}{}^2\right)\left(16D^2-133D+266\right) \\
&-384m_H{}^2\,(s_{12}+s_{14})\,(D-2)-260\,(D-2)\,s_{13}{}^2-176s_{13}\,(s_{12}+s_{14})\,(D-2) \\
&-38\left(s_{12}{}^2+s_{14}{}^2\right)(D-2)+16\,(D-2)\,s_{12}s_{14}+384\,(D-2)\,m_H{}^4 \qquad (2.27) \\
&-8\,(s_{12}s_{23}+s_{14}s_{34})\left(-4+2D+D^2\right)+8\,(s_{14}s_{23}+s_{12}s_{34})\left(D^2+12-6D\right) \\
&+2\left(16D^2+182-91D\right)s_{34}s_{23}\Big] + \frac{2}{s_{34}s_{124}}\Big[s_{12}\,(s_{14}+s_{24})\left(-305D+613+20D^2\right) \\
&+\left(-285D+42D^2+316\right)s_{12}{}^2+24\left(s_{13}{}^2+s_{23}{}^2\right)(-16+5D)+48\,(-16+5D)\,s_{13}s_{23} \\
&-\left(s_{24}{}^2+s_{14}{}^2\right)\left(-553+6D^2+148D\right)-2\,(s_{13}s_{24}+s_{14}s_{23})\left(1083-454D+56D^2\right) \\
&-2\,(s_{13}s_{14}+s_{23}s_{24})\left(-390D+48D^2+955\right)-2\left(-108D+38D^2-41\right)s_{24}s_{14} \\
&-2s_{12}\,(s_{13}+s_{23})\left(52D^2-458D+1091\right)\Big] - \frac{8}{s_{234}s_{124}}\Big[-8\,(s_{12}s_{14}+s_{23}s_{34})\,(D-2) \\
&-6\,(s_{14}s_{23}+s_{12}s_{34})\,(D-2)+4\left(s_{12}{}^2+s_{14}{}^2+s_{23}{}^2+s_{34}{}^2\right)(D-2) \\
&+6\,(s_{12}s_{23}+s_{14}s_{34})\,(D-2)+48m_H{}^2\left(s_{24}+m_H{}^2\right)(D-2)-\left(-98D+11D^2+184\right)s_{24}{}^2 \\
&+2\left(11D^2+148-80D\right)s_{24}s_{13}-\left(11D^2-52D+92\right)s_{13}{}^2\Big] \\
&+\frac{32m_H{}^2}{s_{13}s_{14}s_{123}}\Big[\left(s_{12}{}^2+s_{12}s_{24}+s_{23}{}^2\right)(D-2)-6\,(D-3)\,s_{34}s_{12}+6\,(D-3)\,s_{34}s_{23} \\
&-2\,(D-2)\,s_{12}s_{23}-(D-2)\,s_{23}s_{24}+2s_{24}\,(s_{34}+s_{24})\,(D-2)+4\,(D-2)\,s_{34}{}^2\Big] \\
&-\frac{8}{s_{234}s_{24}s_{134}}\Big[(D-2)\,s_{14}{}^3-24\,(D-2)\,s_{34}s_{12}s_{14}-12\,(D-2)\,s_{12}{}^2s_{13}-8\,(D-2)\,s_{12}s_{13}s_{14} \\
&-7\,(D-2)\,s_{34}{}^2s_{13}-3\left(s_{13}{}^2s_{34}+s_{14}{}^2s_{34}+s_{13}s_{14}{}^2\right)(D-2)-(D-2)\,s_{13}{}^3+3\,(D-2)\,s_{13}{}^2s_{14} \\
&+4s_{12}\left(s_{13}{}^2+s_{14}{}^2\right)(D-2)+16\,(D-2)\,s_{12}{}^3+6\,(D-2)\,s_{34}s_{13}s_{14}+12\,(D-2)\,s_{12}{}^2s_{14} \\
&-4\left(-D+2+2D^2\right)s_{34}s_{12}{}^2+7\,(D-2)\,s_{34}{}^2s_{14}+4\left(4D^2-19D+38\right)s_{34}{}^2s_{12} \\
&+24\,(D-2)\,s_{34}s_{12}s_{13}-\left(8D^2-43D+86\right)s_{34}{}^3\Big] \\
&+\frac{16}{s_{234}s_{12}s_{34}}\Big[-8\left(s_{13}s_{24}{}^2+s_{13}s_{14}s_{23}+s_{14}s_{23}{}^2+s_{13}s_{14}s_{24}\right)(D-2) \\
&+m_H{}^2\,(s_{13}s_{23}+s_{14}s_{24})\,(D+4)\,(D-2)-m_H{}^2\,(s_{13}s_{24}+s_{14}s_{23})\,(D-2)\,(D-12) \\
&+4\left(s_{13}s_{14}{}^2+s_{13}{}^2s_{14}+m_H{}^2s_{14}{}^2+m_H{}^2s_{13}{}^2+s_{14}{}^2s_{23}+s_{13}{}^2s_{24}\right)(D-2) \\
&+8\,(D-2)\,m_H{}^2s_{13}s_{14}-2m_H{}^2\left(s_{24}{}^2+s_{23}{}^2\right)\left(-21D+3D^2+40\right) \\
&+4\left(s_{23}{}^3+s_{13}s_{23}{}^2+s_{14}s_{24}{}^2+s_{24}{}^3\right)\left(-7D+14+D^2\right)-4\left(-7D+D^2+16\right)s_{24}m_H{}^2s_{23}\Big]
\end{aligned}$$

Factorising Singularities

We showed in [arXiv:1011.4867] that for color singlets all possible singularity structures can be factorised using projective scalings.

Example:

$$\int_0^1 \mathrm{dx_1 dx_2 dx_3} \frac{1}{(x_1+x_2+x_3)^{3+\epsilon}}$$

Singularity:

$$x_1 = x_2 = x_3 = 0$$

Projectify: $x_i \to \dfrac{x_i}{1+x_i}$

$$\to \int_0^\infty \mathrm{dx_1 dx_2 dx_3} \frac{[(1+x_1)(1+x_2)(1+x_3)]^{1-\epsilon}}{(x_1+x_2+x_3+2(x_1x_2+x_2x_3+x_3x_1)+3x_1x_2x_3)^{3+\epsilon}}$$

Rescale: $x_1 \to x_1 x_3 \qquad x_2 \to x_1 x_3$

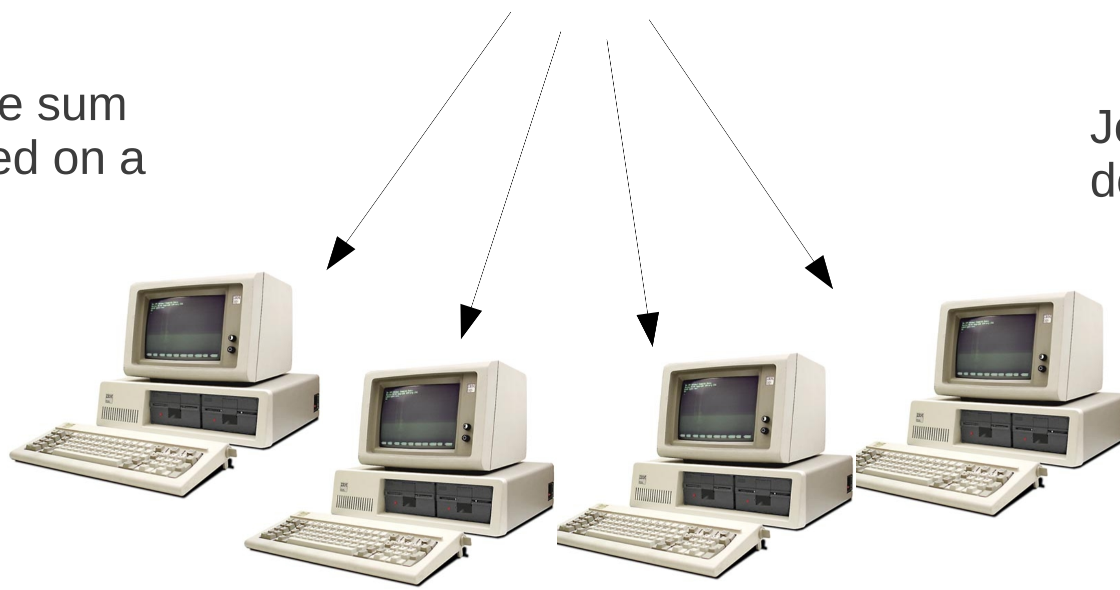
$$\to \int_0^\infty \mathrm{dx_1 dx_2 dx_3}\; x_3^{-1-\epsilon} \frac{[(1+x_1x_3)(1+x_2x_3)(1+x_3)]^{1-\epsilon}}{(x_1+x_2+1+2(x_1x_2x_3+x_2x_3+x_3x_1)+3x_1x_2x_3^2)^{3+\epsilon}}$$

Laurent expansion in the dimensional regulator is then trivial!

This RR method allows for very efficient parallel evaluation

$$\Rightarrow \sigma[J] = \sum_i \int \mathrm{d}\Phi F_i(\{p_i\}) \sum_{\sigma \in G_i} J(\{p_\sigma\})$$

Each term in the sum can be evaluated on a seperate core!



Job management done in Python

Typical runtime to get 1% precisoin on the total inclusive Cross section is about 20 minutes on a Laptop.

Conclusions & Outlook

- Presented eHiXs a new tool for Higgs boson production.
- Presented a method for double real emissions based on factorisation of overlapping singularities using projective scalings and integrand reduction using Groebner basis for residues which respects the symmetry properties of amplitudes .
- Successfully applied the method for Higgs production in gluon fusion and implemented it into eHiXs.
- EhiXs is now in the final stages of testing, and we hope to release it soon to provide a flexible framework for Higgs production.
- Beyond the application to Higgs production at NNLO the integrand reduction technique in conjunction with the factorisation of singularities looks promising.
- It would be very interresting to further understand the universality of these residues and their connection to amplitude factorisation, and ultimately whether there exists an easier way to get to obtain such a representation?